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ON SPACE OF FOUR DIMENSIONS.

BY GEORGE S. FULLERTON.

In the "Quarterly Journal of Science"¹ for April, 1878, appeared an article, by J. C. Friedrich Zöllner, Professor of Physical Astronomy in the University of Leipsic, "On Space of Four Dimensions." The facts which the author thinks prove the actual existence of such a space, or at least make its assumption a reasonable hypothesis, are given in the first volume of his "Scientific Treatises,"² and, after presenting in his article the general argument to prove that the *possibility* of a four-dimensional space is not inconceivable, he cites one of these facts to prove it an actuality.

From the fact that Zöllner's treatises have excited considerable interest and some discussion in Germany, and that a leaning to the belief in a space of four or more dimensions is by no means uncommon, and seems to present a special attraction to those accustomed to mathematical reasonings; and from the additional

¹ Vierteljahrsschrift.

² "Wissenschaftliche Abhandlungen," von Joh. Carl Friedrich Zöllner, Professor der Astrophysik an der Universität zu Leipzig. Erster Band. Leipzig: L. Staackmann, 1878.

fact that the peculiar misconception which underlies the argument presented by Zöllner is specious and oft-recurring—an error into which many have fallen before him, and many more are likely to fall in the future—an analysis of his argument, and a notice of the misconceptions upon which it is based, will not be without interest. Omitting certain sections which are unnecessary to an understanding of the positions taken, his argument, as it stands in the "Quarterly Journal," is as follows:

"In accordance with Kant, Schopenhauer, and Helmholtz, the author regards the application of the law of causality as a function of the human intellect given to man *a priori*—*i. e.*, before all experience. The totality of all empirical experience is communicated to the intellect by the senses—*i. e.*, by organs which communicate to the mind all the sensual impressions which are received at the *surface* of our bodies. These impressions are a reality to us, and their sphere is two-dimensional, acting not in our body, but only on its *surface*.

"We have only attained the conception of a world of objects with three dimensions by an intellectual process. What circumstances, we may ask, have compelled our intellect to come to this result? If a child contemplates its hand, it is conscious of its existence in a double manner: in the first place by its tangibility, in the second by its image on the retina of the eye. By repeated groping about and touching, the child knows by experience that his hand retains the same form and extension through all the variations of distance and positions under which it is observed, notwithstanding that the form and extension of the image on the retina constantly change with the different position and distance of the hand in respect to the eye. The problem is thus set to the child's understanding, How to reconcile to its comprehension the apparently contradictory facts of the *invariableness* of the object, and the *variableness* of its appearance. This is only possible within space of three dimensions, in which, owing to perspective distortions and changes, these variations of projection can be reconciled with the constancy of the form of a body.

"The moment we observe in three-dimensioned space contradictory facts—*i. e.*, facts which would force us to ascribe to a body two attributes or qualities which hitherto we thought could not exist together—the moment, I say, in which we should observe

such contradictory facts in a three-dimensioned body, our reason would at once be forced to reconcile these contradictions.

“I now proceed to apply the higher conception of space to the theory of twisting a perfectly flexible cord. Let us consider such a cord to be represented by *a b*, showing us, when stretched, a development of space in *one* dimension—

(*a* ————— *b*).

If the cord is bent so that during this action its parts always remain in the same plane, a development of space in *two* dimensions will be required for this operation. The following figure may be given to the cord :

(*a* —————  *b*)

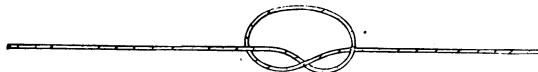
and all its parts, if conceived of infinite thinness, may be considered as lying in the same plane—*i. e.*, in a development of space in *two* dimensions. If the flexible cord, without being broken, has to be brought back into the former figure of a straight line in such a manner that during this operation all its parts remain in the same plane, this can only be effected by describing with one end of the cord a circle of 360°.

“For beings with only *two*-dimensional perceptions these operations with the cord would correspond to what we, with our three-dimensional perception, call a knot in the cord. Now, if a being, limited, on account of its bodily organization, to the conception of only *two* dimensions of space, possessed, nevertheless, the ability of executing, by his will, operations with this cord which are only possible in the space of *three* dimensions, such a being would be able to undo this two-dimensional knot in a much simpler way. Merely the turning over of part of the cord would be required, so that after the operation, when all parts again lie in the same plane, the cord would have passed through the following positions :



“If this consideration, by way of analogy, is transferred to a knot in space of *three* dimensions, it will easily be seen that the tying as well as the untying of such a knot can only be effected

by operations, during which the parts of the cord describe a line of *double curvature*, as shown by this figure:



We three-dimensional beings can only tie or untie such a knot by moving one end of the cord through 360° in a plane which is *inclined* toward that other plane containing the two-dimensional part of the knot. But if there were beings among us who were able to produce by their will four-dimensional movements of material substances, they could tie and untie such knots in a much simpler manner by an operation analogous to that described in relation to a two-dimensional knot."

It will be noticed that the argument here presented by Professor Zöllner is purely analogical. From the supposed experience of a *two-dimensional* being, the objects of whose perception are acted upon by a *three-dimensional* being, he draws an inference to our experience should a being inhabiting space of *four* dimensions act upon the objects which we perceive. Finding, as he thinks, such effects,¹ as one might expect to see under those circumstances, produced in the presence of Dr. Henry Slade, a spiritualistic medium, he infers the existence of four-dimensional beings as agents in their production.

Before taking up the fundamental error in his reasonings, we may take exception to his founding an analogical argument upon a single term. If we, by acting in space of three dimensions, can untie a knot of a certain kind in a manner impossible to one moving but on a surface, it does not follow that a knot of a different kind may be untied in a manner impossible to us acting in space of three dimensions by allowing motion in still another—a fourth dimension. If one knot (*a*), which one man can only untie in one way, may be untied in still another way by another man, it does not follow that another and a different knot (*b*), which the second man can untie in only one way, can also be solved in a new way by a third person. For all we know to the contrary, the second knot may admit of but one solution.

If it be proved, however, that we, acting in *three-dimensional*

¹ *E. g.*, the production of true knots in an endless cord.

space, can untie knots which are not to be untied in a space of *two* dimensions, and if it be also proved that in actual experience knots are tied or untied, which seem to us incapable of solution in a space of *three* dimensions, we *may* suppose that it was done by action in the direction of a fourth dimension, though there also remains open to us as alternative the supposition that it was done by a hitherto undiscovered mode of manipulation in space of three dimensions, or by action in a space of five, six, or any other number of dimensions.

The whole argument lapses, however, when it is shown that the supposed experience of two-dimensional beings—the only *datum* for inference to another term—is a supposition without basis, and arising out of a misconception. The manner in which we acquire our conception of space, according to Zöllner, is this: “The totality of all empirical experience is communicated to the intellect by the senses, *i. e.*, by organs which communicate to the mind all the sensual impressions which are received at the *surface* of our bodies. These impressions are a reality to us, and their sphere is two-dimensional, acting not in our body, but only on its *surface*.” This gives us the idea of a surface. In explaining how we arrive at the idea of the third dimension, or distance, Zöllner follows a similar method to Berkeley, in his “New Theory of Vision,” and refers the idea to the experienced connection of the variable visual appearance with the constant tangible object.

Although Zöllner has followed Berkeley (to whose essays he refers in his article), it is evident that he has not understood the force of his reasonings. The statement that the impressions of sense “act at the surface of our bodies,” and that through them we gain the idea of a surface (two-dimensional space) before we know space in a third dimension, is a double misunderstanding. The impressions of sense, if by this phrase sensations are designated, are not felt primarily at the surface of our bodies, and are only localized after a long visual and tactful experience of the organism—an experience which implies as its outcome a knowledge of space in its three dimensions.

The sensations given us by contact with objects would not at first have *position* or *coexistence* in *space*, but only succession, or coexistence in *time*, until after the fixing of the relations of visual and tactful sensations—they could be localized. After that they

would, of course, suggest the space-idea on being themselves awakened—which would be a going back, however, from conclusion to premises. There is no necessary connection between any particular sensation and the part of the body to which we relegate it. It is not felt *in* the part, and all localization of sensation is a result of experience and observation. Before the idea of the organism, as extended, no sensation could be regarded as spatially *out of* another.

Again. The idea that we know a surface before we know the third dimension is untenable. A surface, as we know it, implies the idea of distance—it presupposes the knowledge of a third dimension. In the latter part of his essay on "Vision" (§§ 155–158) Berkeley speaks of this. In the inquiry concerning what knowledge a spirit endowed with the power of vision, but without the sense of touch, would have of geometry, after denying that he would have any knowledge of a solid, or quantity of three dimensions, he continues: "and, perhaps, upon a nice inquiry, it will be found he cannot even have an idea of plane figures any more than he can of solids, since some idea of distance is necessary to form the idea of a geometrical plane, as will appear to whoever shall reflect a little on it." "I must confess it seems to be the opinion of some very ingenious men that flat or plane figures are immediate objects of sight, though they acknowledge solids are not; and this opinion of theirs is grounded on what is observed in painting, wherein (say they) the ideas immediately imprinted in the mind are only of planes variously colored, which, by a sudden act of the judgment, are changed into solids; but, with a little attention, we shall find the planes here mentioned as the immediate objects of sight are not visible, but tangible planes. For when we say that pictures are planes, we mean thereby that they appear to the touch smooth and uniform. But then this smoothness and uniformity, or, in other words, this planeness of the picture, is not perceived immediately by vision; for it appeareth to the eye various and multiform." A similar error to Zöllner's was that made by Sir William Hamilton in his lecture on the relations of sight and touch to extension. In inquiring whether extension is the object of sight, he argues as follows:¹ "All parties are, of course, at one in regard to the fact

¹ "Metaphysics," New York, 1880, p. 385.

that we see color. Those who hold that we see extension admit that we see it only as colored; and those who deny us any vision of extension make color the exclusive object of sight. In regard to the first position, all are, therefore, agreed. Nor are they less harmonious in reference to the second—that the power of perceiving color involves the power of perceiving the differences of colors. By sight we, therefore, perceive color, and discriminate one color—that is, one colored body—one sensation of color from another. This is admitted. A third position will also be denied by none—that the colors discriminated in vision are, or may be, placed side by side in immediate juxtaposition; or, one may limit another by being superinduced partially over it. A fourth position is equally indisputable—that the contrasted colors, thus bounding each other, will form by their meeting a visible line, and that, if the superinduced color be surrounded by the other, this line will return upon itself, and thus constitute the outline of a visible figure."

It is evident that, in saying that the colors discriminated in vision may be "placed side by side in immediate juxtaposition," Sir William is using language which implies a knowledge of distance. The planes to which he refers are not purely visual. To vision alone we must allow some discrimination between the colors, that they may become representative of tactful differences, but what that discrimination would be to one who had never enjoyed the sense of touch we have no means of knowing. It certainly would not be like our present knowledge of the differently colored planes. In his "Review of Sir W. Hamilton's Philosophy," Mr. Mill has pointed out with clearness the misconceptions in this supposed argument of Sir William's, and has justly complained that the position, line, and figure of which it treats are not the objects of pure vision.

Our idea of space is not derived from visual sensation alone, nor from tactful impressions alone, but is, so to speak, the nearly simultaneous representation, by a few visual symbols, of a multitude of successive tactful impressions. The visual symbols, before they are interpreted in tactful impressions, can give no true idea of space any more than a sight of the letters and words can give the ideas contained in a book to one who has not learned to read. But the sign and the thing signified may become so closely

connected by long experience that one may easily fall into error as to the share of the whole impression to be attributed to the one element and to the other.

Our knowledge of a surface, or space of two dimensions, therefore, implies a knowledge of distance, which is necessary to the interpretation of the visual symbols, and without which they would be meaningless. And we have, consequently, no idea what would be the conception of space of a "two-dimensional" being, nor how he would be affected by a manipulation of the twisted cords. Reasoning from our experience of a surface, and the movements of cords on a surface, to that of such a being, is unwarrantable. We do not know what would be his idea of a line, a surface, or a knot—in short, any analogical argument based upon his experience is based upon something to us totally unknown and inconceivable.

If it be objected that all this relates to a visual knowledge of extension and not to a tactful, and that, though the idea of distance, or extension in a third dimension, be necessary to the former, it may not be to the latter; I answer that our idea of space is a complex of the two, the interpretation by a general formula of many particulars, whereby, if we may accept the results of the observations of Platner on the blind,¹ the idea of simultaneity or coexistence is substituted for that of succession.

The idea, therefore, of spatial extension must be very different in one who has never enjoyed the sense of sight from what it is in one who has, and we could not argue from the experience of such to our own.

Moreover, it is evident that Zöllner does not refer at least a knowledge of tri-dimensional space to tactful experience alone, but refers it to an attempt to reconcile our apparently contradictory visual and tactful experience, so that our knowledge of the third dimension at least would have reference to vision. And if it be denied that we can gain any idea of a surface from tactful impressions before a localization of sensations, it is incumbent upon Zöllner to show how they would ever give rise, taken alone, to the idea of a surface.

But, even granting that we consider a purely tactful knowledge

¹ Quoted by Hamilton, "Metaphysics," New York, 1880, p. 389.

of space, we have no reason to believe that there could be knowledge of a surface prior to a solid, or independently of knowledge of extension in a third direction; though here we are reasoning largely in the dark, as we cannot tell what may be the notion of *direction* in the mind of a blind person, or how it compares with our own, which has always reference to visual experience. We have no idea whatever what would be the conception of space in the mind of a blind "two-dimensional" being; but we may at least assume that, whatever might be the nature of his conception, it would have little or nothing in common with our idea of a surface.

Consequently, the argument from the twisted cords is wide of the mark, and the whole analogical argument from the experience to two-dimensional beings, the objects of whose perception are acted upon by us from the direction of a third dimension, to that of three-dimensional beings in their relation to four-dimensional, is an analogy drawn in fact from *our* knowledge of a surface, and *our* knowledge of a solid, to something inconceivable, and shows a misconception of the force of the reasoning contained in the "New Theory of Vision."

DANTE'S "INFERNO."

BY SUSAN E. BLOW.

To know how hard the wind is blowing one must sail against the wind. To measure the force of a stream one must swim against its current. That the tendencies of any given age may be comprehended, they must be surveyed from the standpoint of an age different in its habits of thought. Drifting with his generation, the individual cannot gauge its strength, and sees neither the direction in which it moves nor the goal towards which it tends.

We live in an age which is rapidly losing the consciousness of sin. Equally alien to our feeling are the physical self-scorgings of the mediæval saint and the spiritual agony of the Puritan.